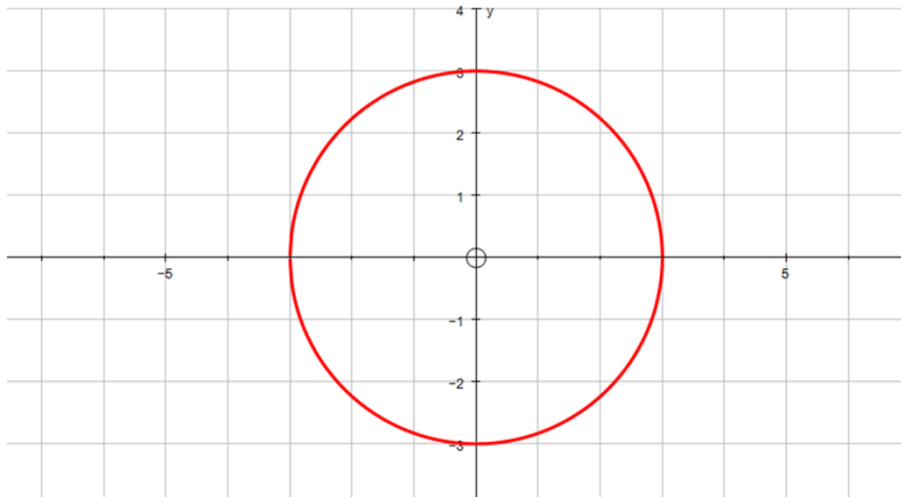


Further Mathematics

Conics

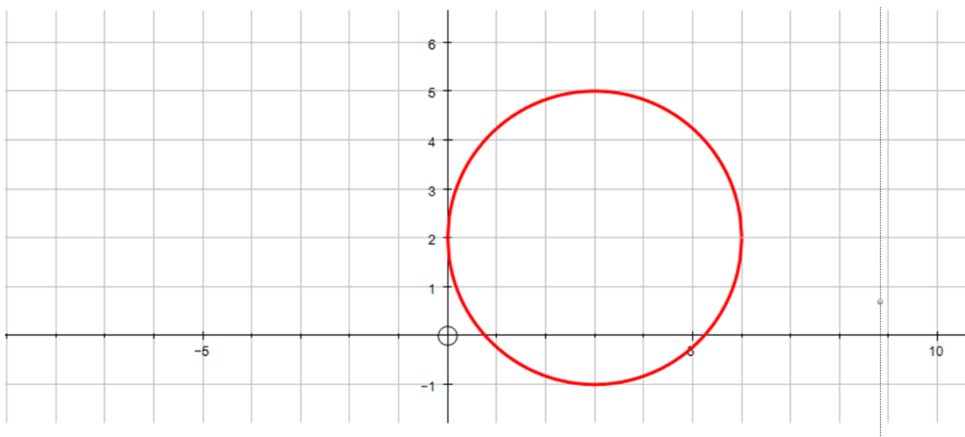
For this work you need a graphing application. Go to <https://www.desmos.com/calculator> for a free version.

In Desmos, draw this circle:



What equation did you use?

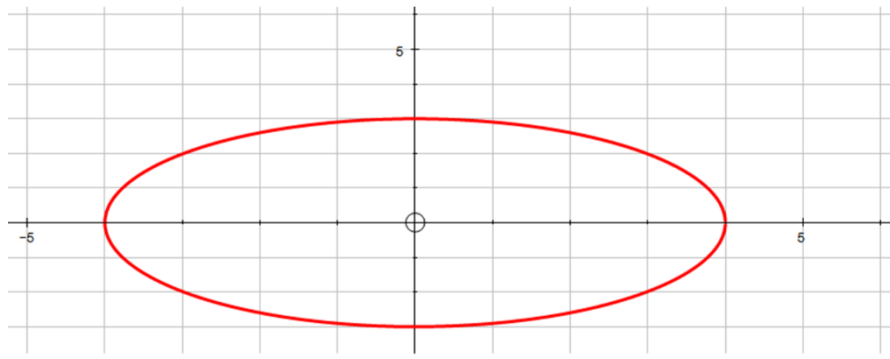
Now move this circle, so the centre is now (3, 2)



What is the new equation?

How would we describe the movement mathematically?

Now draw:

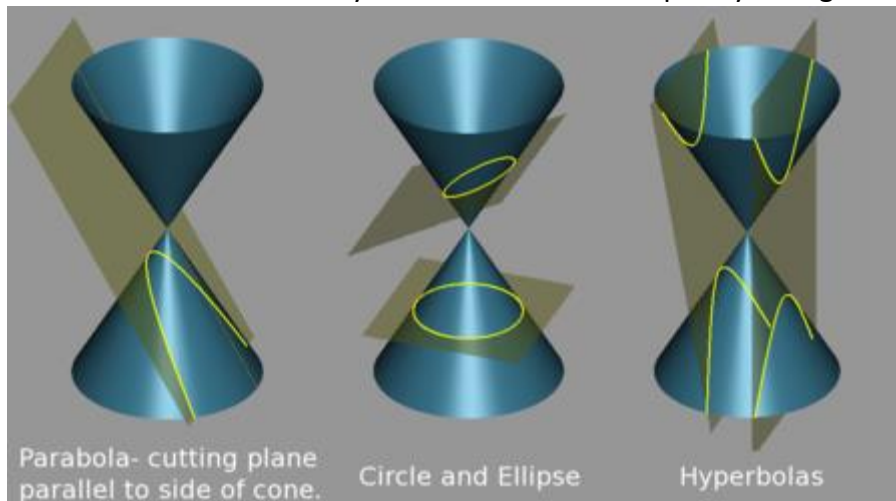


What is this shape called?



Now draw a different one.

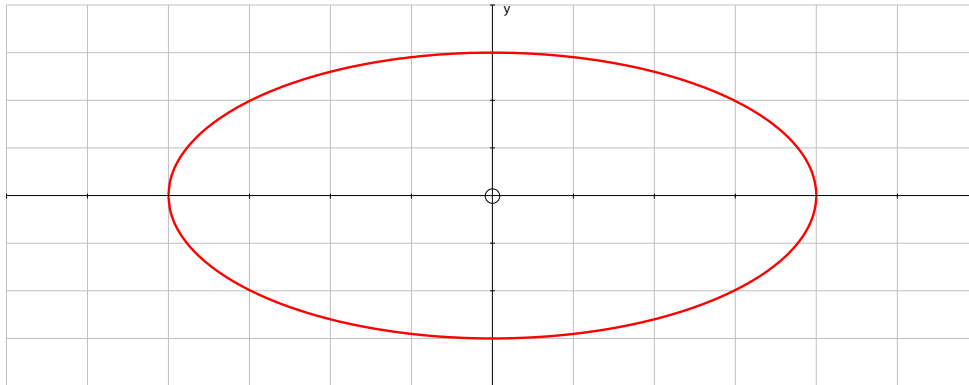
These curves are called conics. This is because you can create their shapes by slicing through cones.



You are now going to investigate conics using Desmos. In the standard form of the equation, choose numeric values for a , b or c to see what impact varying these values has.

Ellipse

Standard form of equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



Where does the graph cross the x axis?

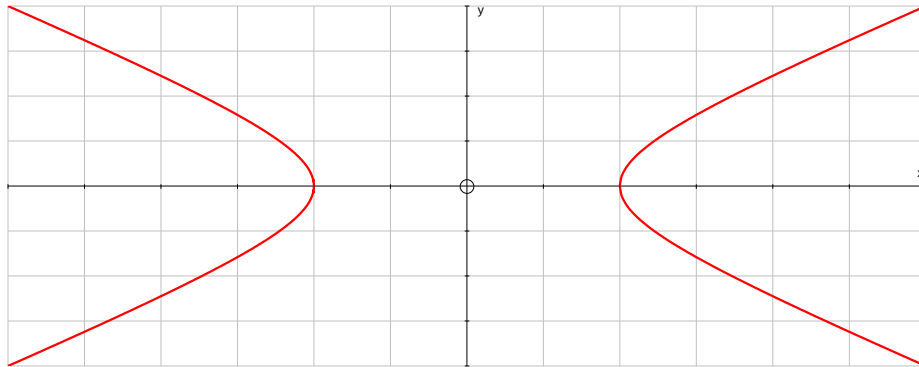
Where does the graph cross the y axis?

What is the equation of this ellipse?

What would be the equation of the ellipse the same shape as this one but with centre (3,2)?

Hyperbola

Standard form of equation: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

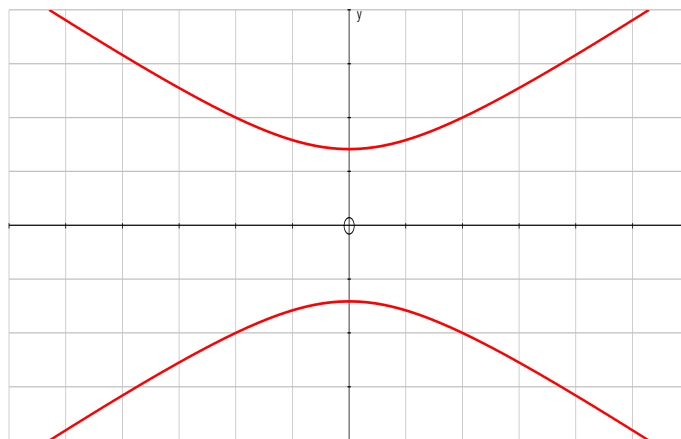


Where does the graph cross the x axis?

Where does the graph cross the y axis?

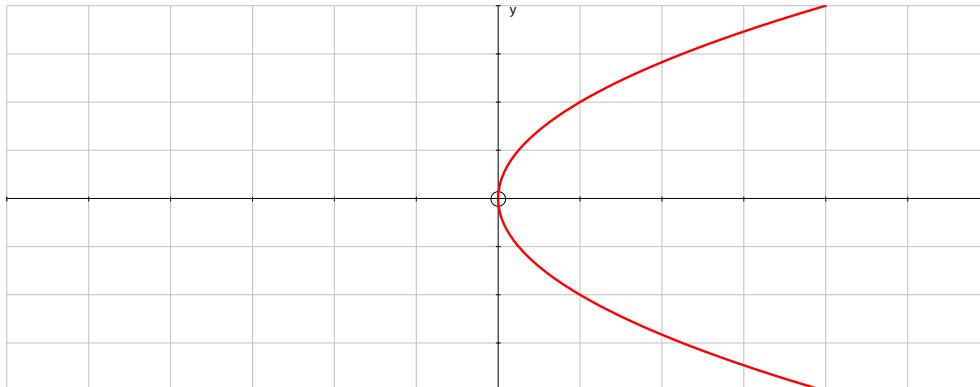
What is the equation of this hyperbola?

What would be the equation of the hyperbola that crosses the y axis at 4 and -4?



Parabola

Standard form of equation: $y^2 = 4ax$

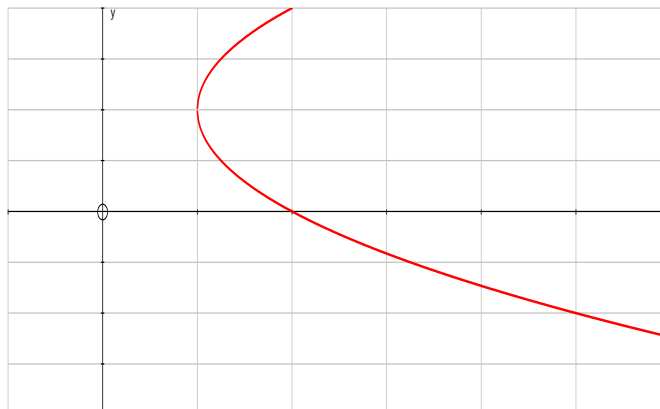


Where does the graph cross the x axis?

Where does the graph cross the y axis?

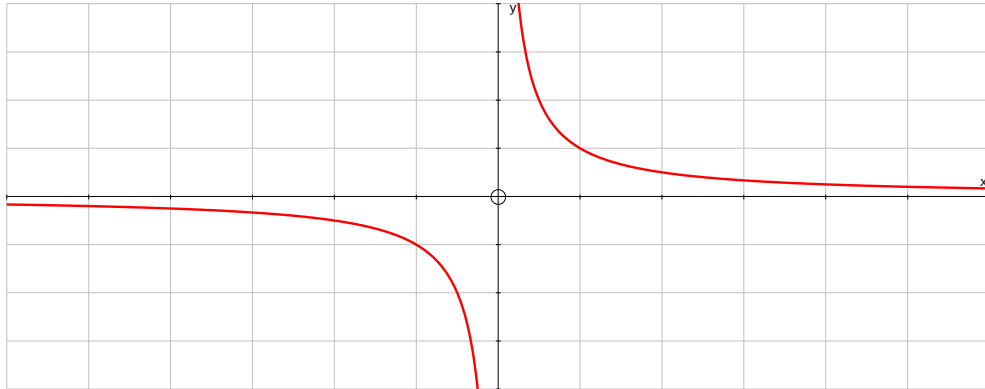
What is the equation of this parabola?

What would be the equation of the parabola the same shape as this one but with “turning point” (3 , 2) ?



Rectangular Hyperbola

Standard form of equation: $xy = c^2$

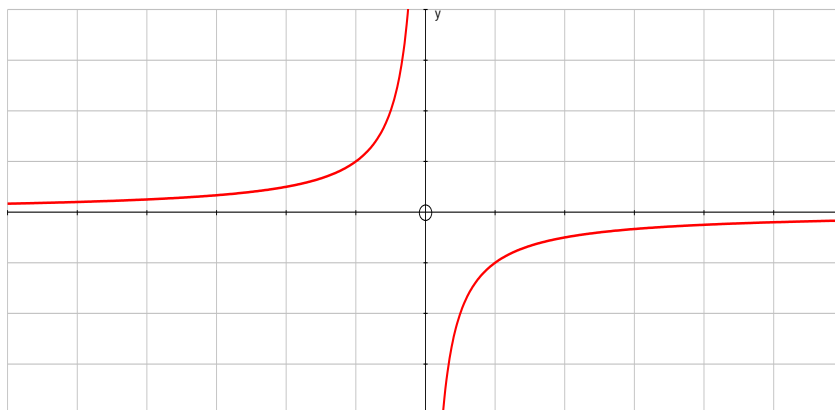


Where does the graph cross the x axis?

Where does the graph cross the y axis?

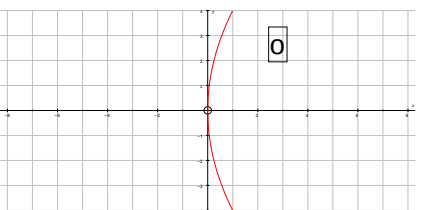
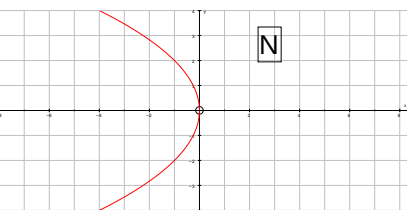
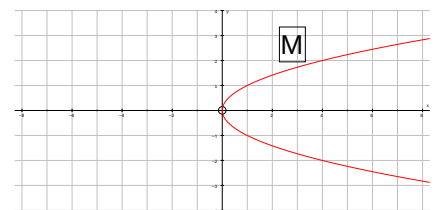
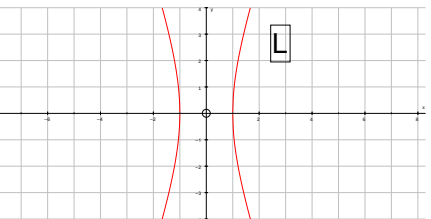
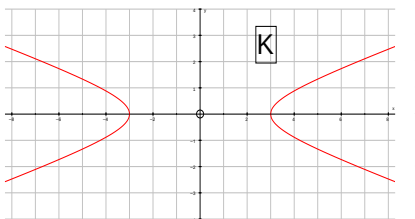
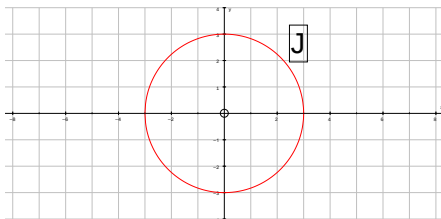
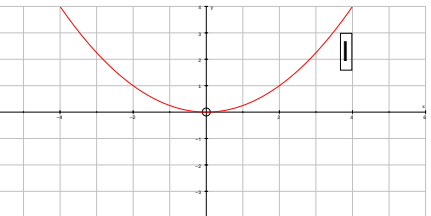
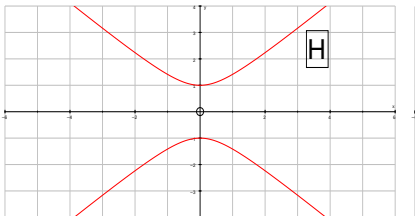
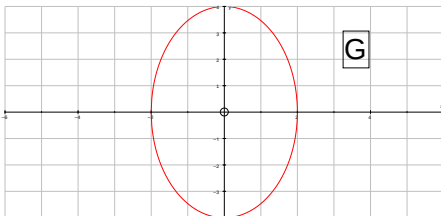
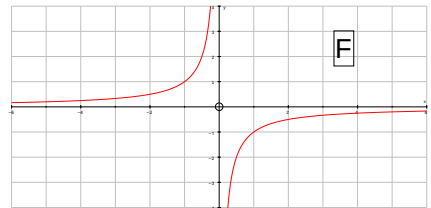
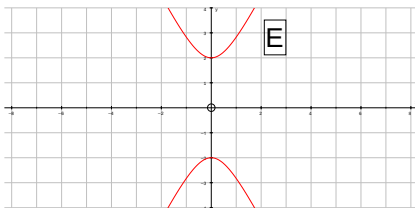
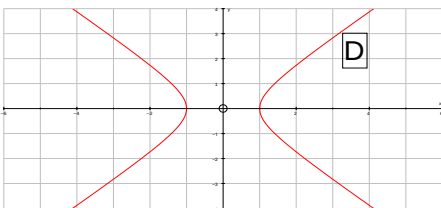
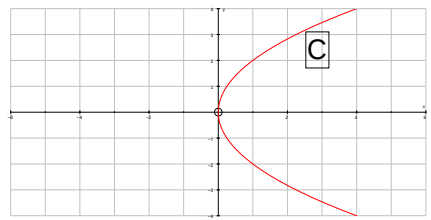
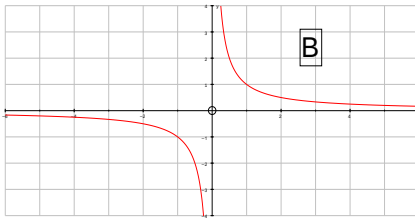
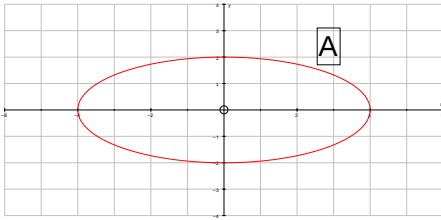
What is the equation of this rectangular hyperbola?

What would be the equation of the rectangular hyperbola in the other two quadrants?



Identifying Conics

Now match the graph to its equation, and write the number of the correct equation in the table below. Try to do this by yourself first, then check your answers using Desmos.



1	$\frac{y^2}{4} - x^2 = 1$	5	$x^2 - \frac{y^2}{9} = 1$	9	$xy = -1$	13	$x^2 - y^2 = 1$
2	$y^2 = 4x$	6	$y^2 - x^2 = 1$	10	$\frac{x^2}{9} + \frac{y^2}{9} = 1$	14	$\frac{x^2}{9} - y^2 = 1$
3	$y^2 = x$	7	$y^2 = 16x$	11	$xy = 1$	15	$x^2 = 4y$
4	$\frac{x^2}{4} + \frac{y^2}{16} = 1$	8	$\frac{x^2}{16} + \frac{y^2}{4} = 1$	12	$y^2 = -4x$		

Table for solutions:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O

How did you get on?

Check here when you have finished.

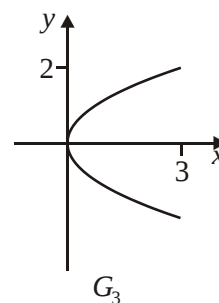
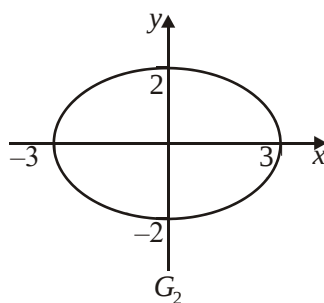
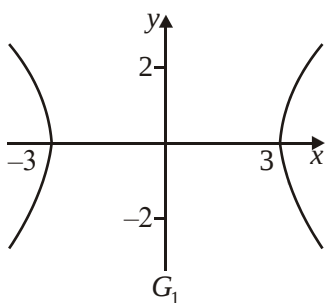
Now try these questions:

1. Sketch the graph of $\frac{x^2}{16} + \frac{y^2}{49} = 1$, marking the values of the intercepts with the coordinate axes.

2. Sketch the curve with equation $y^2 = 8x$.

3. (a) The graph of $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is one of those sketched below.

Identify whether it is graph G_1 , G_2 or G_3 .



(b) On separate sets of axes, sketch the graphs of:

(i) $\frac{(x+3)^2}{9} - \frac{y^2}{4} = 1$;

(ii) $\frac{y^2}{9} - \frac{x^2}{4} = 1$.

4. A curve has equation

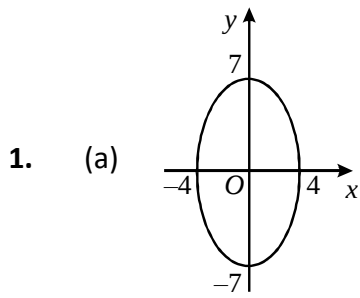
$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

(a) Sketch the curve, showing the coordinates of the points of intersection with the coordinate axes.

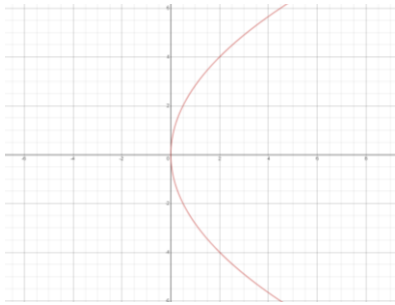
(b) Calculate the y -coordinates of the points of intersection of the curve with the line $x = 1$. Give your answers in the form $p\sqrt{2}$, where p is a rational number.

(c) The curve is translated one unit in the positive x direction. Write down the equation of the curve after the translation.

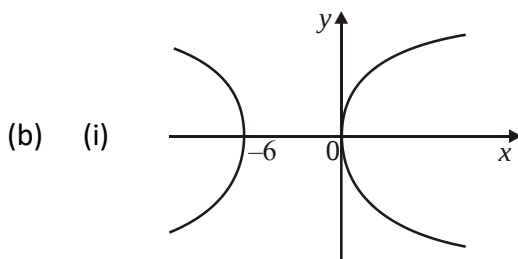
Answers



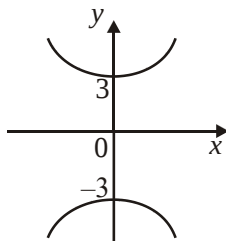
2.



3. (a) G1



(ii)



4. (a) Correct shape. Coordinates $(\pm 3, 0)$, $(0, \pm 2)$ shown

(b) Attempt to solve $\frac{1}{9} + \frac{y^2}{4} = 1$

At least one correct root

Allow decimals; allow $\sqrt{\frac{32}{9}}$

$$y = \pm \frac{4}{3}\sqrt{2}$$

(c) Eqn is $\frac{(x-1)^2}{9} + \frac{y^2}{4} = 1$